3/11 Things you should know

PID Intuition

- what the terms are
- know what k_p, k_i, k_d do to a step response
- intuitive PID tuning process (section 2.1 in 2/9 notes)

Modeling

- scalar ODE → state space ODE
- transfer function model: output for a pure sinusoid input. (bottom of page 7, 2/9 notes)

Numerical Integration

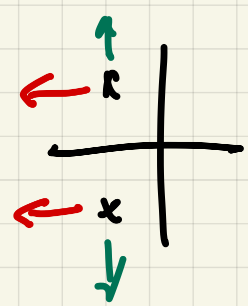
- two types of "approximation"
- errors accumulate

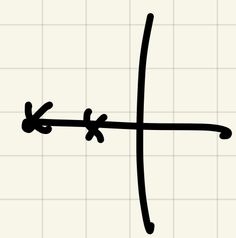
Nonlinear State Space $\dot{x} = f(x, u)$

- find eq. pts., know eq. pt. definition
- linearize about an eq. pt. x_e

Linear State Space $\dot{x} = Ax + Bu \quad y = Cx$

- eq. pt. facts (come back to)
- stability tests for $x_e = 0$ (don't need to know Jordan form)
- stability definitions (asymptotically stable, stable, unstable)
- controllability definition and test
- eigenvalue placement
  - linear state feedback $u = -Kx \Rightarrow \dot{x} = (A - BK)x$
  - when can you do it?
  - use MATLAB place (or python equivalent)
- Complex plane intuition
  - second order step responses
  - dominant eigenvalues





- LQR
  - know $J(x, u)$
  - tuning Q, R

~ use MATLAB lqr

Observers

- definition and test
- observer error dynamics $e = x - \hat{x} \Rightarrow \dot{e} = (A - KC)e$
- eigenvalue placement for observers
  - use MATLAB command
- putting it all together
  - how do you connect controller/observer/tracking

Eq. pts. of linear systems $\dot{x} = Ax + Bu$

$$Ax_e = 0 \qquad A \in \mathbb{R}^{n \times n} \qquad x \in \mathbb{R}^n$$

$x_e = 0$ is always an eq. pt.

<u>Linear Algebra</u>

$$\text{Null}(A) = \{ x \in \mathbb{R}^n \mid Ax = 0, \; x \neq 0 \}$$

Column(A) all linear combinations of the columns of A

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}$$

$$\text{Column}(A) = \{ x_1 a_1 + x_2 a_2 + x_3 a_3 \mid x_i \in \mathbb{R}, \; i = 1,2,3 \}$$

$$= \{ \sum_{i=1}^{n} x_i a_i \mid x_i \in \mathbb{R}, \; i = 1, 2, \ldots, n \}$$

$$x_1 a_1 + x_2 a_2 + x_3 a_3 = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= Ax$$

$\text{column}(A) = \{ Ax \mid x \in \mathbb{R}^n \}$

$Ax = b$ solve for x

a solution exists if $b \in \text{column}(A)$

if $\text{column}(A) = \mathbb{R}^n$ the solution always exists

$\Rightarrow$ A is invertible

if $\text{column}(A) \subset \mathbb{R}^n$ but not equal to $\mathbb{R}^n$

$\Rightarrow$ a solution might not exist

$\Rightarrow$ A is not invertible

The part of $\mathbb{R}^n$ not covered by $\text{column}(A)$ is the null space $\text{Null}(A)$

$$\text{column}(A) + \text{null}(A) = \mathbb{R}^n$$

$$\text{column}(A) \perp \text{null}(A)$$